ECE4722 Spring2020: Power Electronics Course Project

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## Derivation of the Small Signal Model of a Non-Ideal Buck Converter

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Figure 1. An implementation of an Ideal Buck Converter

Given the ideal buck converter shown in Figure 1, a Non-Ideal Buck Converter may be modelled by considering the conduction loss of the MOSFET and/or diode using the respective conduction resistances and , where the switch is modelled with a small resistance when on, or it is an open circuit when off. The system can be represented as a small-signal model with state variables, and , as the current through the inductor and voltage across the capacitor, and input variables and as the input voltage and output current, while the output variable is , the output voltage. Using the standard state space format with matrices A’, B’, C’, and D’, the small signal model is derived in the following figures, first by using Kirchhoff’s voltage and current laws to derive the equations for the state variables, then by converting the equations into their state space format to achieve the large-scale model, and finally averaging both states of the system and deriving the small signal model.

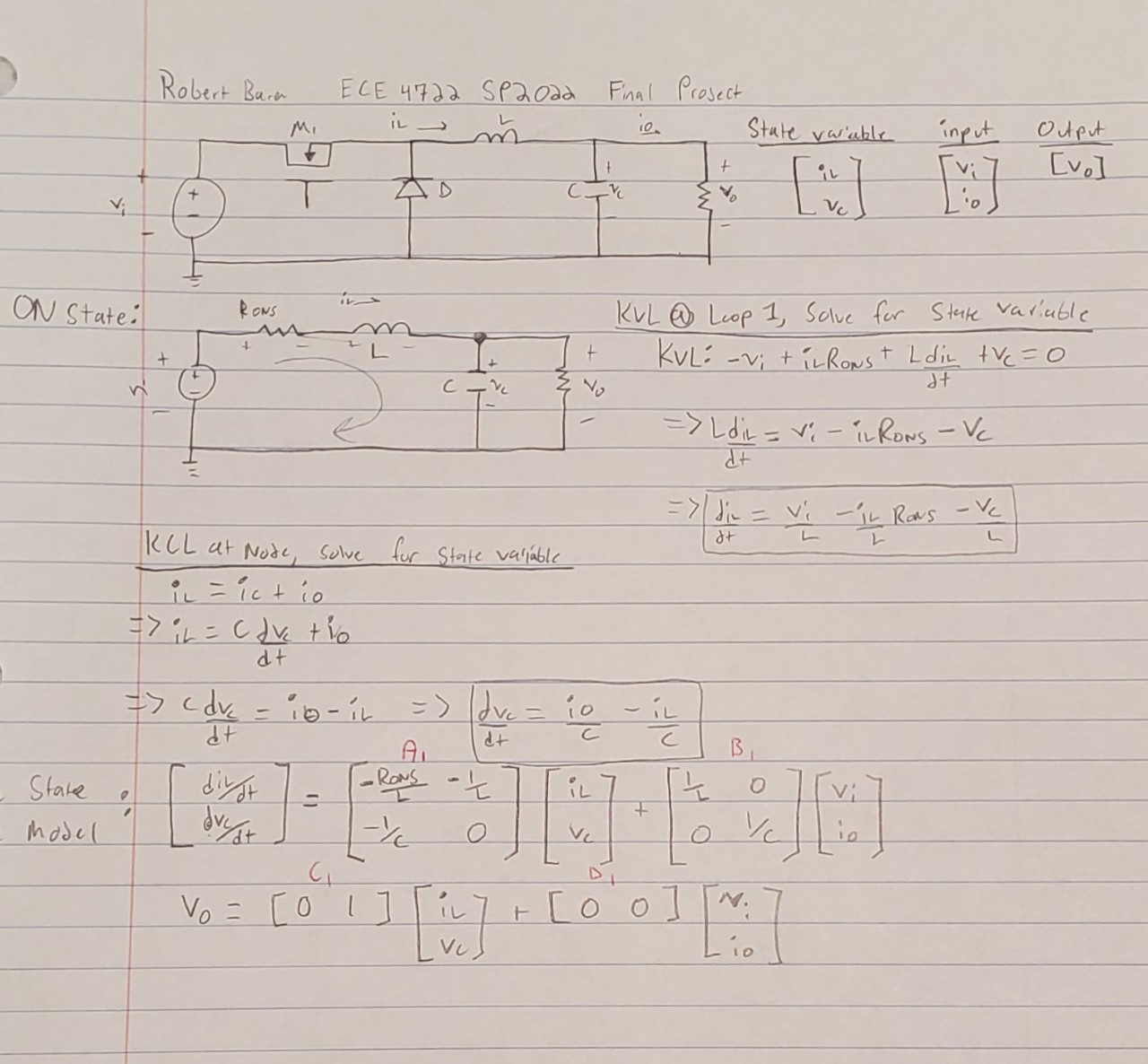


Figure . Derivation for Buck Converter switch "on" state

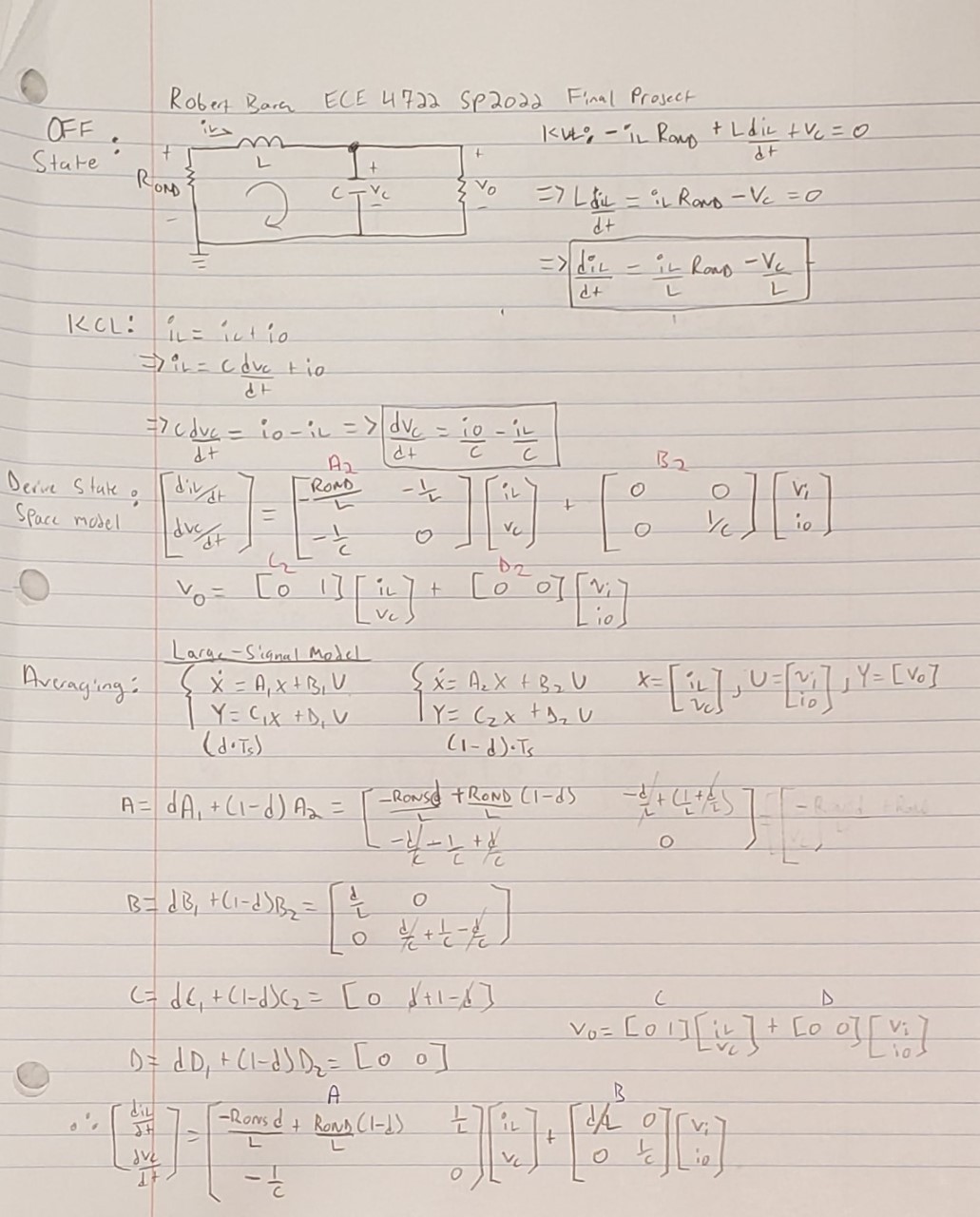
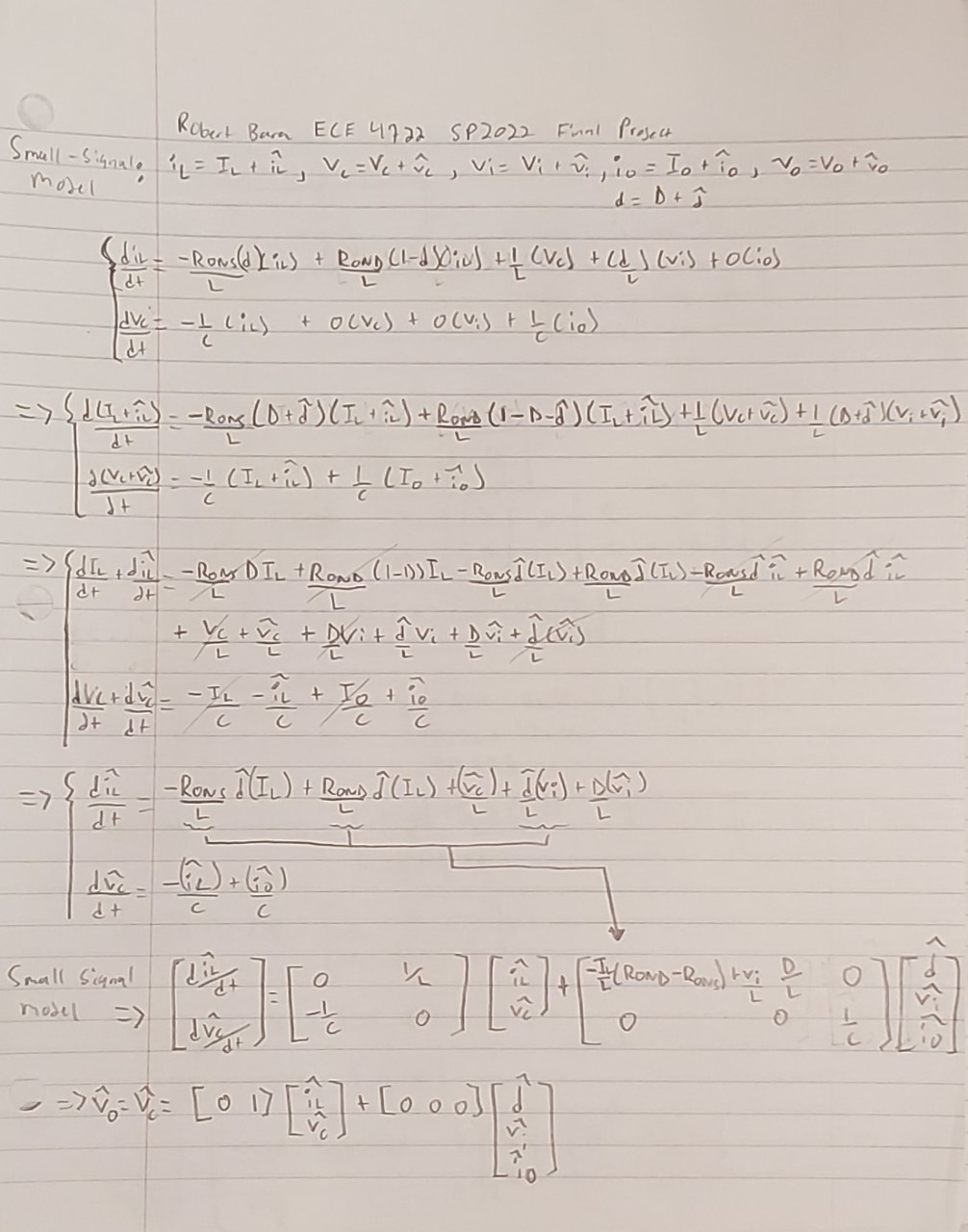


Figure . Derivation of Buck Converter switch "off" State, and State Space Averaging

Figure . Derivation of the Small Signal Model for the Non-Ideal Buck Converter

## Deriving the Small-Signal Model for an Ideal Buck Converter

Using the Non-Ideal Buck Converter Small-Signal Model, the Ideal Buck Converter Small Signal Model can be derived by setting the perturbations of input variables and to zero, so now only the duty cycle is considered as the input variable. This derivation is summarized by the following hand calculation,

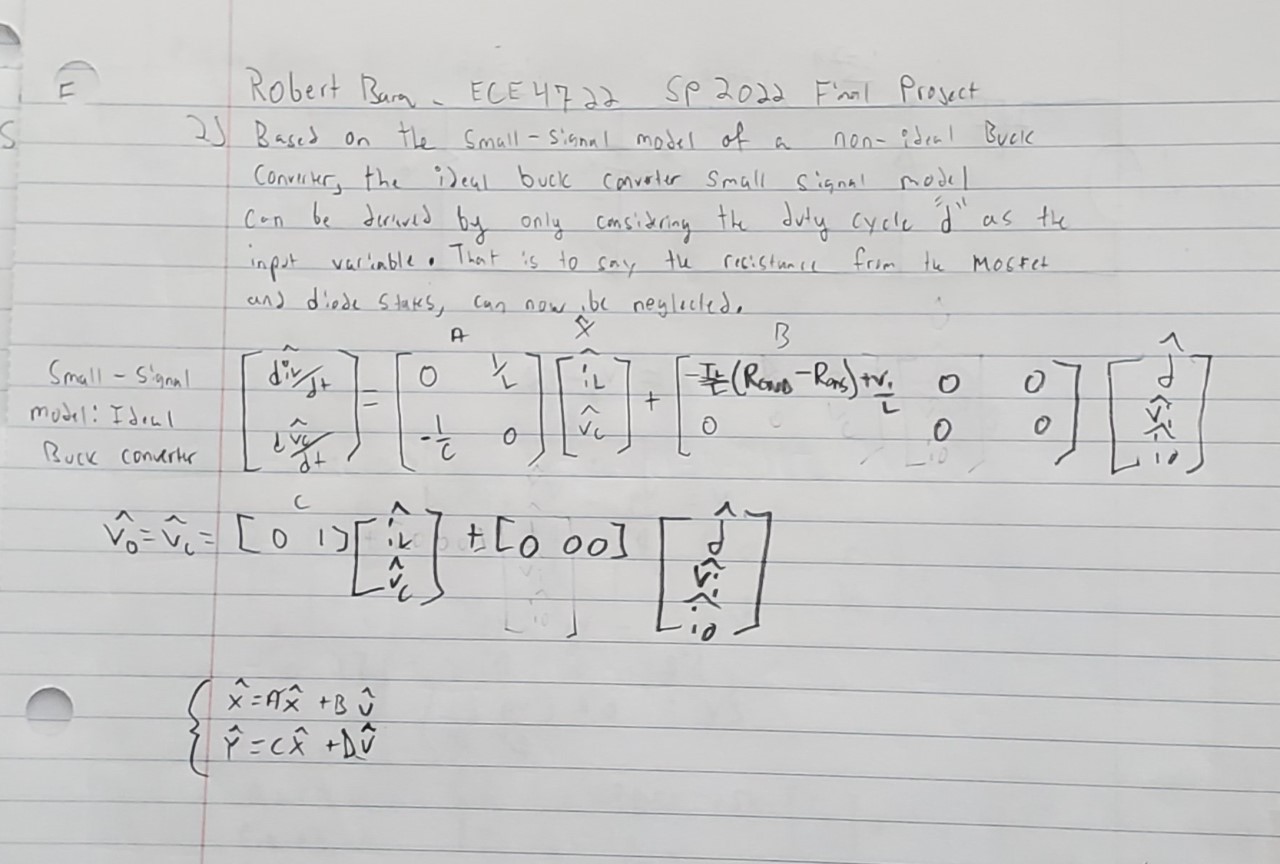


Figure . Derivation of the Small-Signal Model for an Ideal Buck Converter

## 3. Frequency Response of an Ideal Buck Converter

From the small signal model derived in the previous section, the following parameters are given: and a MATLAB script seen below was programmed to plot the frequency response of the Ideal Buck Converter for a duty cycle with inputs of and over a sampling frequency of .

### MATLAB Script

clear all;clc; %#ok<\*CLALL>

syms s

%Define Variables

Vi=20;IL=2;Io=2;C=60e-6;L=400e-6;

Vc=0; Rons=0.1;Rond=0.001;

% Frequency range of 0.5Hz to 200kHz - in rad/s

fs = (0.5:100:200e3).\*2\*pi;

Ts=1./fs;

% Plotting options for figures

% Bodeoptions

opts = bodeoptions;

opts.Grid = 'on';

opts.FreqUnits = 'Hz';

opts.PhaseUnits = 'deg';

%% Calculating when duty cycle is 20%

d=.20;

%Small signal model matricies for buck converter

A1=[0 1/L;-1/C 0 ];

B1=[(-(IL/L)\*(Rond-Rons)+(Vi/L))\*d; 0];

C1=[0 1];

D1=[0];

%Get the state-space representation of the system

sys1=ss(A1,B1,C1,D1);

%% Calculating when duty cycle is 40%

d=.40;

%Small signal model matricies for buck converter

A2=[0 1/L;-1/C 0 ];

B2=[(-(IL/L)\*(Rond-Rons)+(Vi/L))\*d; 0];

C2=[0 1];

D2=[0];

%Get the state-space representation of the system

sys2=ss(A2,B2,C2,D2);

%% Calculating when duty cycle is 60%

d=.60;

%Small signal model matricies for buck converter

A3=[0 1/L;-1/C 0 ];

B3=[(-(IL/L)\*(Rond-Rons)+(Vi/L))\*d; 0];

C3=[0 1];

D3=[0];

%Get the state-space representation of the system

sys3=ss(A3,B3,C3,D3);

%% Plot each figure

figure(1), %For 20% duty cycle

bode(sys1,fs,opts)

figure(2), %For 40% duty cycle

bode(sys2,fs,opts)

figure(3), %For 60% duty cycle

bode(sys3,fs,opts)

Examining the resulting bode plots generated by the script is as follows:

The bode plot for a duty cycle of ,

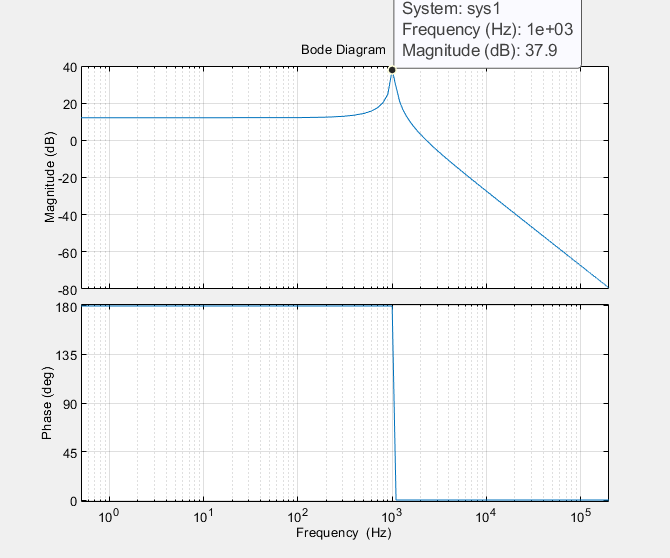


Figure . Frequency response when duty cycle is 20%

Increasing the duty cycle to

Chart

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Figure . Frequency response when duty cycle is 40%

Increasing the duty cycle to .

Chart, line chart

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Figure . Frequency response when duty cycle is 60%

## 4. Simulink Modelling of a Non-Ideal Buck Converter

Utilizing the same circuit parameters, a Simulink model can be developed for the non-ideal buck converter using a Proportional controller and a Proportional Integral controller to achieve an output voltage of . The differences between both controllers are seen in the following subsections.

### P Control for Non-Ideal Buck Converter

The Simulink model for the Non-Ideal Buck Converter is shown below, notice the controller is the function ,

Diagram, schematic

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Figure . Simulink Model for Non-Ideal Buck Converter with P Control

By setting the output reference voltage to , and initiating a step change at , the output voltage waveform is shown on scope 3 from to as follows, where when setting proportional gain the output voltage fluctuates between approximately to and steps down to approximately to at . Since the Buck Converter is Non-Ideal, the system suffers from conduction losses due to and , and the voltages are will not be exactly or due to the small ripple voltage loss, this will happen regardless of how the controller is implemented. Examining the output scope for a Non-Ideal Buck Converter with control it appears that the waveform suffers noise regarded as steady-state error, that is amplified by the proportional gain factor which is the only variable within a Proportional control transfer function.

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Figure . Figure 7. Output Voltage Waveform for P Control

The initial voltage’s fluctuations are consistent between to as seen when the scope is zoomed in before the switching state at .

Calendar

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Figure . Initial state of Output Voltage Waveform

The system then steps down the voltage and stabilizes at where it fluctuates between .

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Figure . Stepped down final voltage ripples

### PI Control for Non-Ideal Buck Converter

The Simulink model for the Non-Ideal Buck Converter is shown below, notice the controller is the function ,

Diagram, schematic

Description automatically generated

Figure . Simulink Model of Non-Ideal Buck Converter with PI Control

By setting the output reference voltage to , and initiating a step change at , the output voltage waveform is shown on scope 3 from to as follows, where the output voltage begins at approximately and steps down to at . Compared to the controlled system, this output is a lot more stable since the gain variable is the gain integrated by integrator , is now added to the system through the transfer function, therefore the system now accounts for the past stepped values and attempts to remove any steady state error over time. This system was tuned by setting and . While increasing the proportional gain would reduce the rise and fall time, it will also increase the error. Increasing the integral gain could also lead to an increase in overshooting oscillations, therefore a compromise results in the following simulation results. This will be seen further when attempting to tune the boost converter in section 5. The output waveform is as follows,

Chart

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Figure . Output Voltage Waveform for PI Control

The initial voltage of with little to no oscillations before the switch at approximately 0.5s,

Chart, histogram

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Figure . Initial state of Output Voltage Waveform

Stepping down after 0.5s and stabilizing to a range between at due to ripples is shown below,

Graphical user interface, application, Word

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Figure . Stepped down output voltage

## 5. Simulink Modelling of a Non-Ideal Boost Converter

While Buck Converters are utilized in power electronics to step down and regulate a DC-DC voltage, the opposing circuit the Boost Converter, regulates DC-DC voltage by stepping up the voltage. Similarly, a Buck/Boost Converter is also a possibility to allow engineers to either step up or step down a desired voltage. At its fundamentals, a Boost Converter can be achieved by simply changing the where the switch (in this case the MOSFET and DIODE) is wired within the circuit. This can be seen by examining the possible switch positions and their impact based on the path of current,

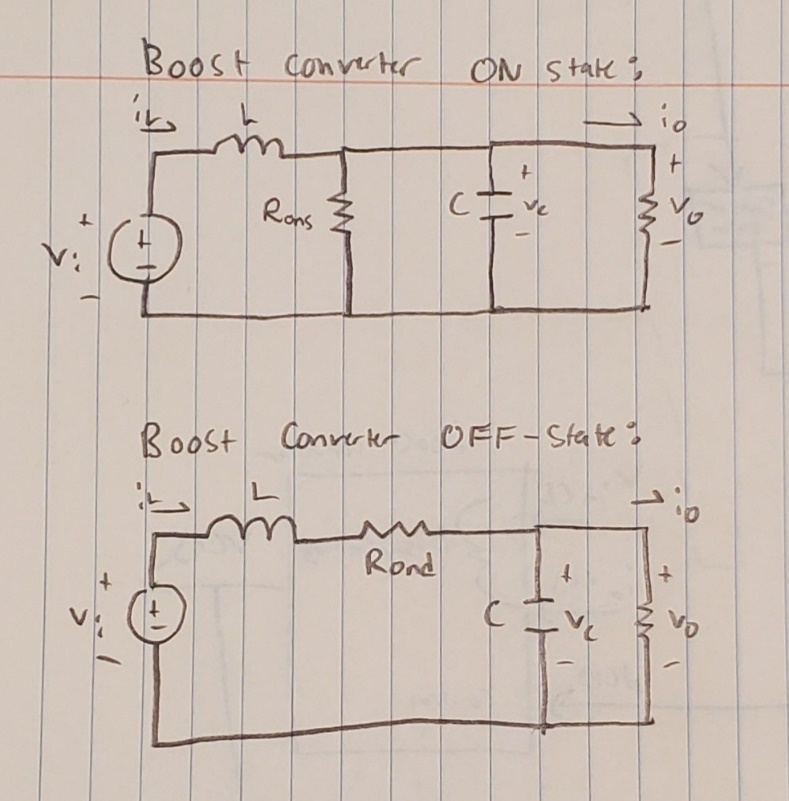


Figure 18. On and Off States for a Non-Ideal Boost Converter

Taking the Non-Ideal Buck Converter simulated in the previous section, Simulink models for a Boost Converter can be achieved and implemented with or control as seen in the following subsections. For the Boost Converter example, the output voltage will now be stepped up from the input voltage of to and output of .

### P Control for Non-Ideal Boost Converter

For a Non-Ideal Boost Converter, the circuit is rearranged to let the MOSFET and Diode act as a switch for the two ideal switching positions seen in the previous figure.

Diagram

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Figure . Simulink Model of Non-Ideal Boost Converter with P Control

Using Simulink’s controller as shown in the figure above, initially setting the proportional gain does yield an increase however this increase does not reach and stabilizes around therefore the controller needs to be tuned.

Chart, line chart

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Figure . Scope of output waveform for Non-Ideal Boost Converter with P control

However, since the proportional controller will only amplify the signal and its errors, trying to achieve an output voltage of is not feasible without making the system unstable. Increasing will decrease the rise time of the system, which is wanted, however the range of the output voltage’s ripples will not increase to approximately Using Simulink’s to tune the controller, I explored this and show that a proportional gain is the maximum value the system can handle without becoming extremely unstable.

Chart, line chart, scatter chart

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Figure . Output Waveform when kp=0.0222

Examining the waveform before the switch, the boost controller captures the initial within a reasonable error, oscillating between to .

Chart

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Figure . Output Waveform Ripples Before Voltage is Stepped Up

The voltage then steps up almost instantly at . Zooming in on the ripples after the voltage is stepped up, we can examine that the system stabilizes with ripples oscillating between to

Graphical user interface

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Figure . Output Voltage Waveform after Switching Sequence

Increasing the proportional gain any higher will result in the harmonics ranging closer to the desired voltage of , however the oscillations will swing below the initial voltage of , therefore one could argue that proportional gain higher than will be unstable. An example of this is shown below when tuning the controller to

Chart, histogram

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Figure . Output Voltage swings between 17V to 38V, this is an unstable system

### PI Control for Non-Ideal Boost Converter

For a Boost Converter using a proportional integral controller, the added integral control will allow for some elimination of the steady state error since the transfer function of the PI controller is now This transfer function is the same as it was for the buck converter, which as stated earlier utilizes the integral control variable to attempt to remove any steady-state error over time.

Diagram

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Figure . Non-Ideal Boost Converter with PI control

Knowing the steady-state error can be removed due to the integral control, the controller was tuned similarly to the previous section. Setting to be again, while setting to be , ensures a fast rise time without adding creating too much overshoot in the overall system’s output voltage ripples.

Chart

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Figure 27. Output Voltage Waveform on Scope with Ripples. Kp=0.01, ki=10

Examining the voltage before the switch has a slight ripple within to V,

Chart, line chart, scatter chart

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Figure .Output Waveform Ripples Before the Voltage is Stepped Up.

Stepping up the voltage, the system stabilizes relatively fast due to the small proportional gain. Zooming in on the voltage after the waveform is stable results in an oscillation between to , which is a much more acceptable range of error than the controller implementation.

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Figure . Outputted Stepped Up Voltage waveform with ripples